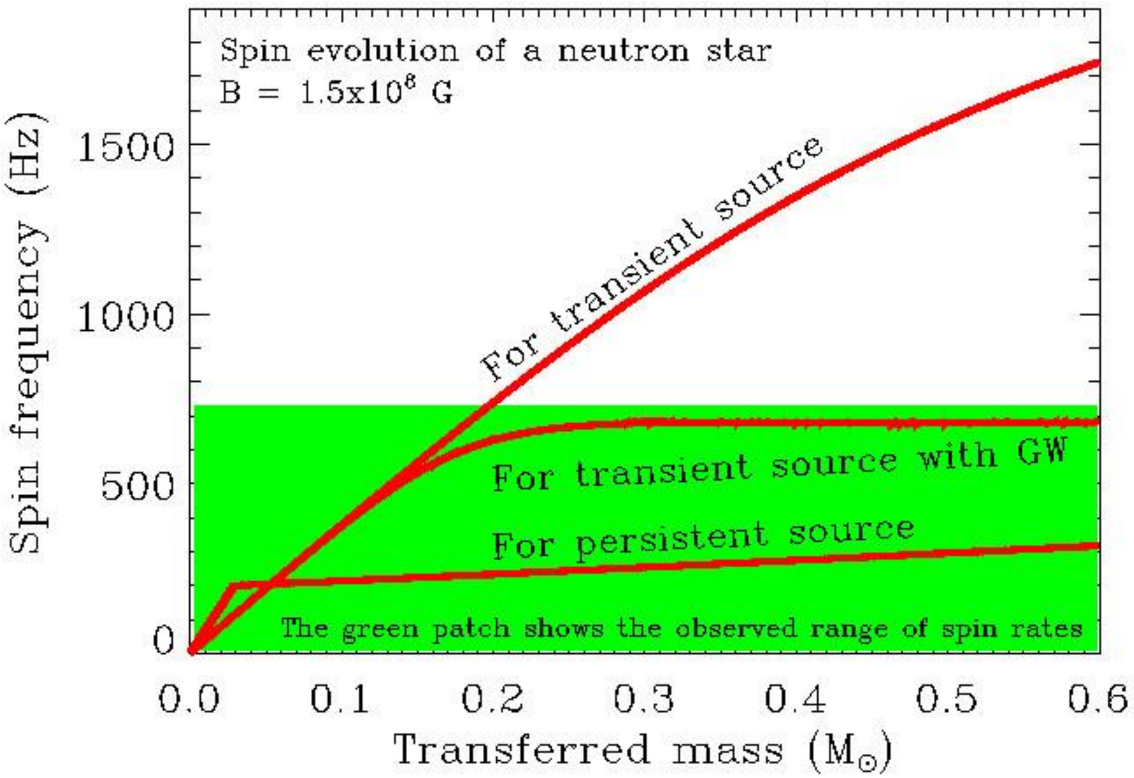
Do some millisecond pulsars emit gravitational waves?



Sudip Bhattacharyya

Tata Institute of Fundamental Research, Mumbai, India

Background and problem

Millisecond pulsars are a type of fast-spinning (~ a few hundred Hz) neutron stars. They are spun up by angular momentum transfer via mass accretion in low-mass X-ray binary (LMXB) systems.

However, we have not observed any neutron star with spin frequency much higher than about 700 Hz.

Figure on the right: →

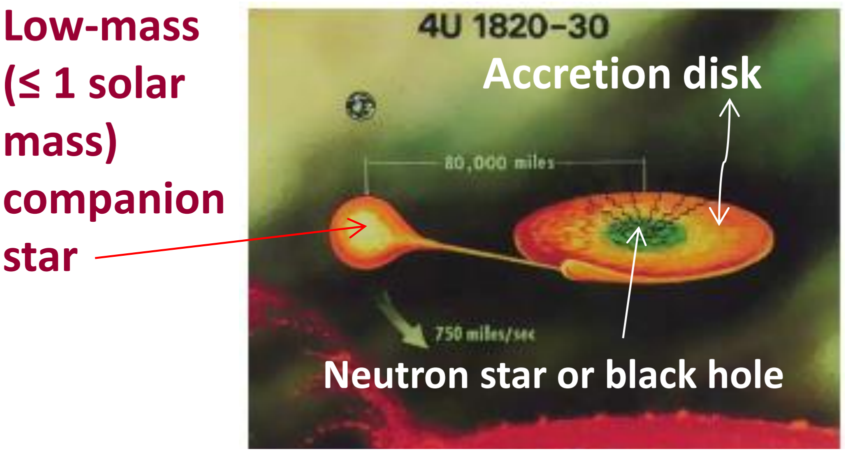
ν_{high} < 730 Hz (99% confidence) (Chakrabarty et al. 2003, Papitto et al. 2014).

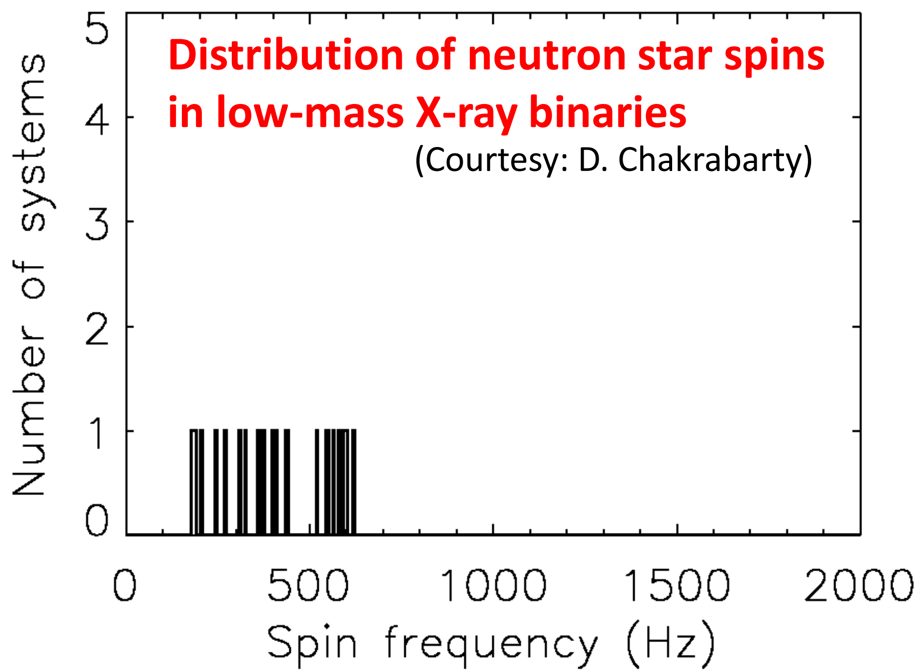
Not a sensitivity issue in the observations.

Fastest known radio pulsar is PSR J1748-2446ad (Ter 5) at 716 Hz.

Low-mass X-ray binary (LMXB)

(Artist's impression)





Background and problem

- We have not observed any neutron star with spin frequency much higher than about 700 Hz.
  (Submillisecond pulsars are evidently relatively rare, if they at all exist, because we have not observed any so far.)

- Neutron star mass-shed limit (or break-up limit), which should be much higher, cannot explain the observed upper limit of ms pulsar spin rates.

What stops accreting pulsars from spinning up beyond ~700 Hz?

What limits the spin rates of millisecond pulsars: suggested solutions

Two suggested solutions.

(1) Interaction between the accretion disk and the neutron star magnetosphere and the resulting neutron star spin equilibrium.

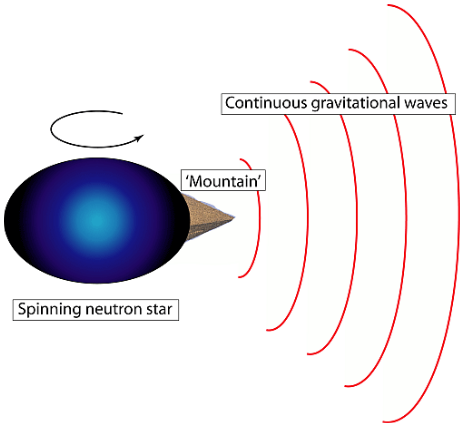
(e.g., Ghosh & Lamb 1979; White & Zhang 1997; Lamb & Yu 2005; Patruno, Haskell, & D'Angelo 2012)

- Cutoff frequency set by distribution of accretion rates, magnetic field strengths, and magnetospheric coupling parameters.

(2) Spin-down due to gravitational waves from the neutron star.

GW torque scales as ~ ν^5 . For a given neutron star ellipticity, GW torque will be negligible until a critical frequency is reached, and will then cut in over a narrow spin range.

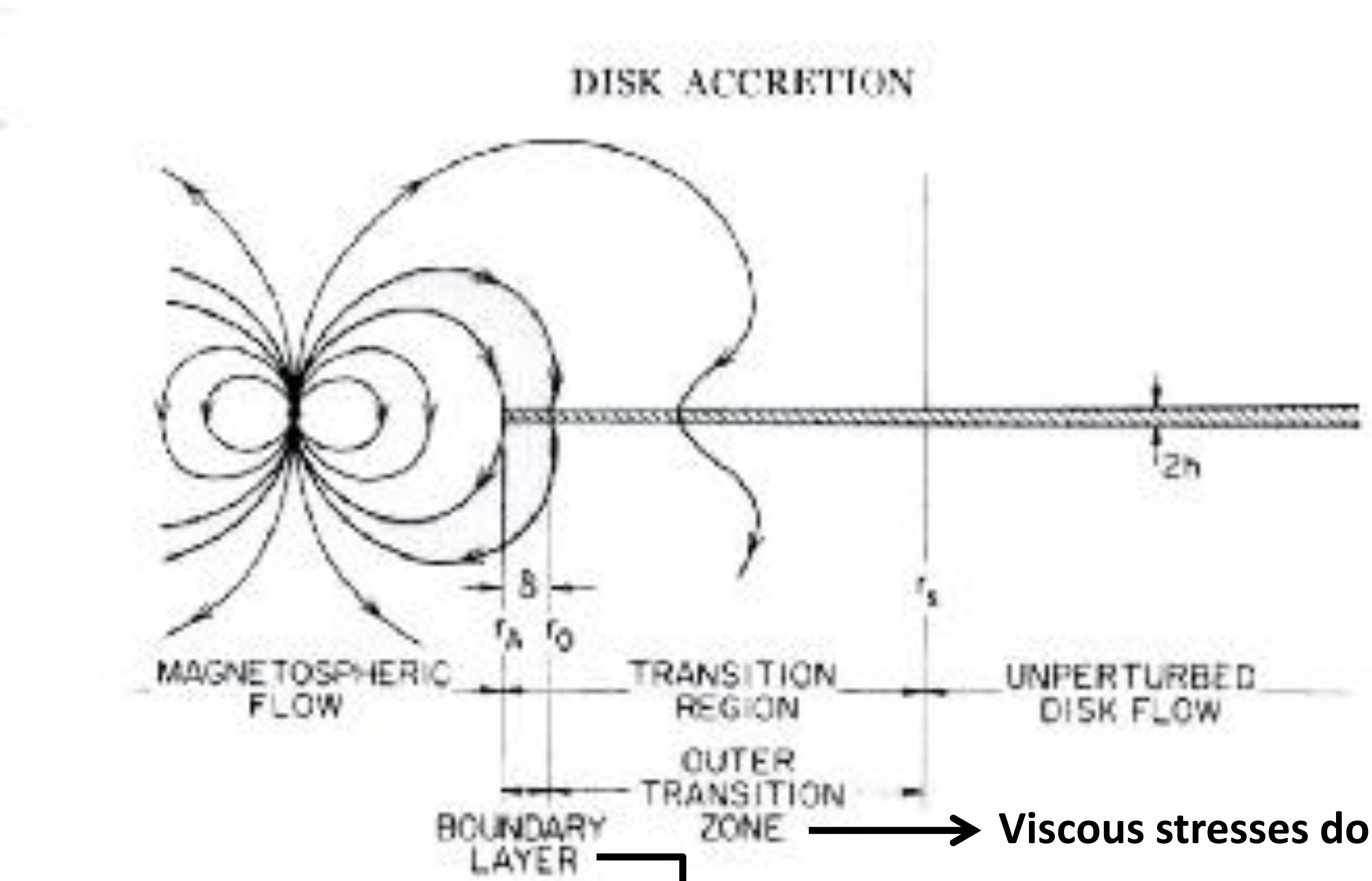
The first mechanism is more conventional.
So first we try to understand if disk-magnetosphere interaction can limit spin rates. This study anyway provides insight of the physics of the accretion through magnetosphere.



If the first mechanism cannot explain the spin rate limit: possible implication: continuous gravitational waves.

Courtesy: LSC

Disk-magnetosphere interaction



→ **Viscous stresses dominate**

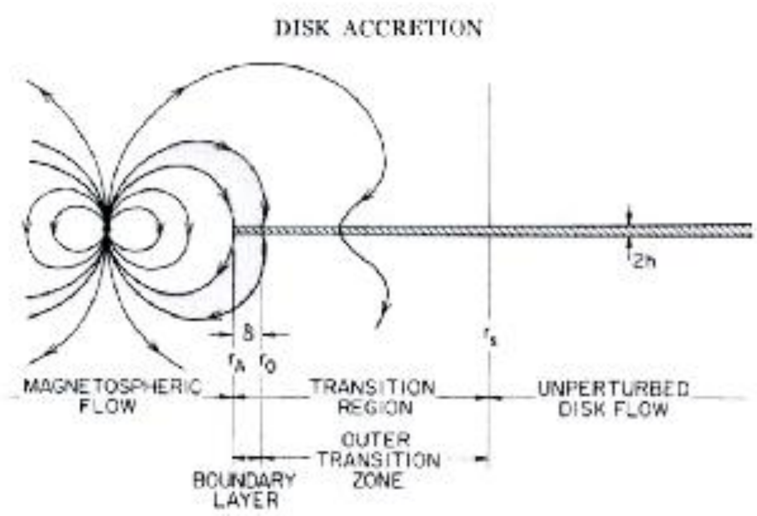
→ **Magnetic stresses dominate ($\delta << r_A$)**

Ghosh and Lamb (1978)

Disk-magnetosphere interaction

Magnetospheric radius
r_m = ξ.[(B^2R^6)/($\dot{M}$. (2GM)$^{0.5}$)]$^{2/7}$, where the accretion disk stops. (**ξ ~ 0.5-1.4**).

Corotation radius **r_{co} = [(GM)/(2πν)^2]$^{1/3}$**, where the Keplerian spin frequency is equal to the NS spin frequency**.**



For **r_m < r_{co} ⇒ positive torque (for example, $\dot{M}$.[GMr_m]$^{0.5}$) ⇒ spin up ⇒ r_{co} decreases**

For **r_m > r_{co} ⇒ negative torque and propeller effect ⇒ spin down ⇒ r_{co} increases**

So a self-regulated mechanism operates:
r_{co} tends to r_m . At r_m = r_{co} : no torque ⇒ spin equilibrium.

Spin equilibrium frequency ν_{eq} = 3000 Hz . $\xi^{-3/2}$.$B_8^{-6/7}$. $R_6^{-18/7}$.$M^{5/7}$.($\dot{M}$/ $\dot{M}_{Edd}$)$^{3/7}$

Disk-magnetosphere interaction

Spin equilibrium frequency ν_{eq} = 3000 Hz . $B_8^{-6/7}$. $R_6^{-18/7}$.$M^{5/7}$.$(\dot{M}/\dot{M}_{Edd})^{3/7}$

Spin equilibrium frequency is the maximum spin frequency a neutron star can attain via accretion-induced spin-up.

So for reasonable ranges of parameter values (given that the average $\dot{M}$ is only a small fraction of $\dot{M}_{Edd}$ for most sources), the disk-magnetosphere interaction alone could explain the observed upper limit of neutron star spin frequency.

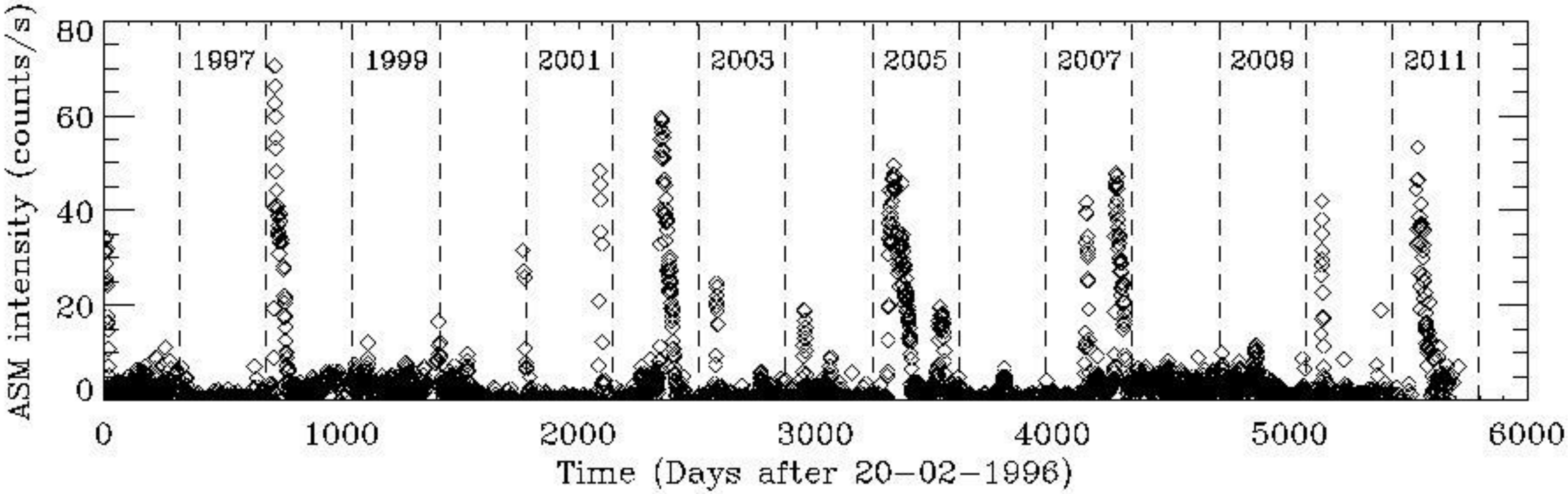
Hence, It was proposed that a spin-down due to gravitational waves may not be required, and the spin equilibrium frequency can possibly explain the observed upper limit of ms pulsar spin rates.

But is it true?

The effect of transient accretion was ignored earlier, which we consider in our work.

New Development: Effect of Transient Accretion

4U 1608-522



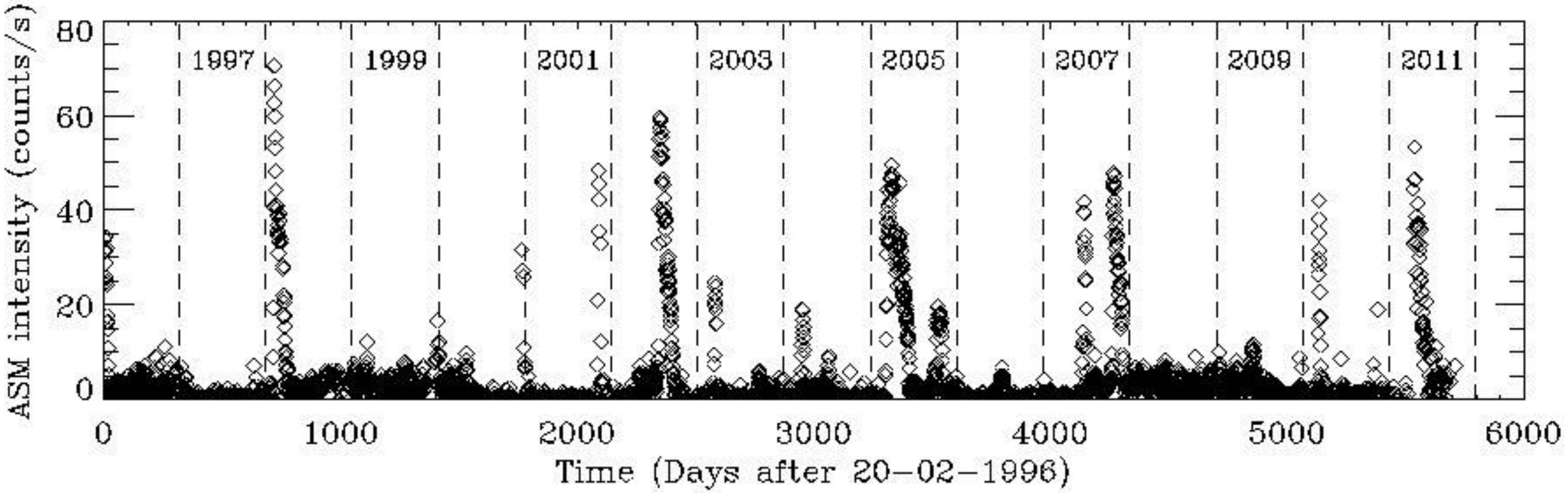
Most of the neutron star LMXBs are X-ray transient sources. Moreover, almost all the X-ray ms pulsars (in fact all known accreting ms pulsars) are transients. So the effect of transient accretion on the ms pulsar spin evolution should be considered.

But pulsar spin evolution is traditionally computed by assuming quasi-persistent accretion at the long-term average accretion rate.

SB & D. Chakrabarty, 2017, ApJ, 835, 4: How is spin evolution affected if transient accretion is treated explicitly?

Transient neutron star LMXBs

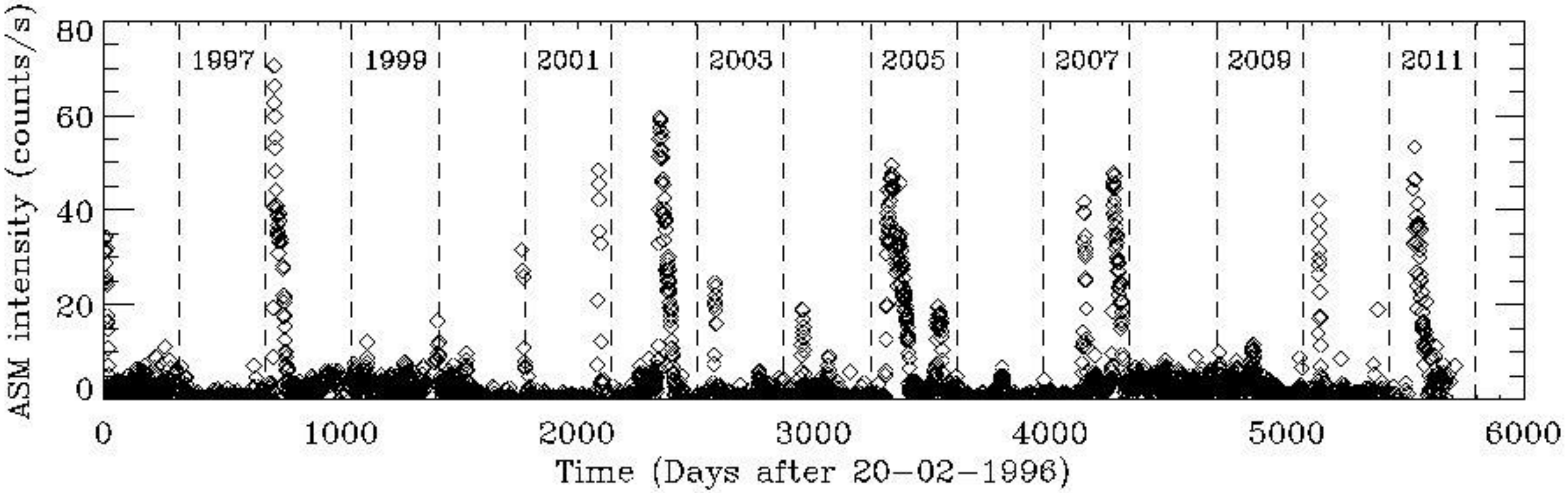
4U 1608-522



Does transience make any difference in the spin equilibrium condition and frequency?

Transient neutron star LMXBs

4U 1608-522



Does transience make any difference in the spin equilibrium condition and frequency?

Yes, a crucial difference.

Because, for transients, r_m drastically changes, as $\dot{M}$ ($\propto r_m^{-7/2}$) evolves by several orders of magnitude in an outburst cycle. Therefore, except for one r_m value, the spin equilibrium condition ($r_m = r_{co}$) is not satisfied throughout the outburst.

So what should be the spin equilibrium condition for a transient source?

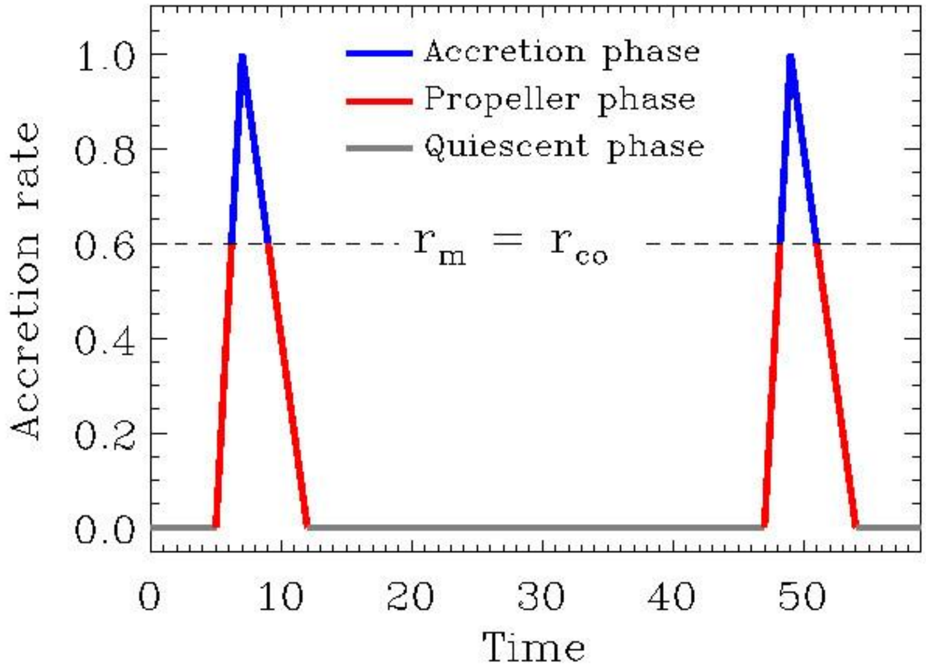
Numerical results

Transient neutron star LMXBs

We numerically compute the spin evolution of a neutron star through a series of outbursts for various sets of parameter values.

SB & D. Chakrabarty, 2017, ApJ, 835, 4

Series of outbursts: three phases of each outburst cycle



Magnetospheric radius
$r_m = \xi . [(B^2R^6)/(\dot{M}. (2GM)^{0.5})]^{2/7}$,
where the accretion disk stops.

Corotation radius $r_{co} = [(GM)/(2\pi\nu)^2]^{1/3}$,
where the Keplerian spin frequency is equal to the NS spin frequency.

SB & D. Chakrabarty, 2017, ApJ, 835, 4

Three different regimes of accretion were theoretically identified.

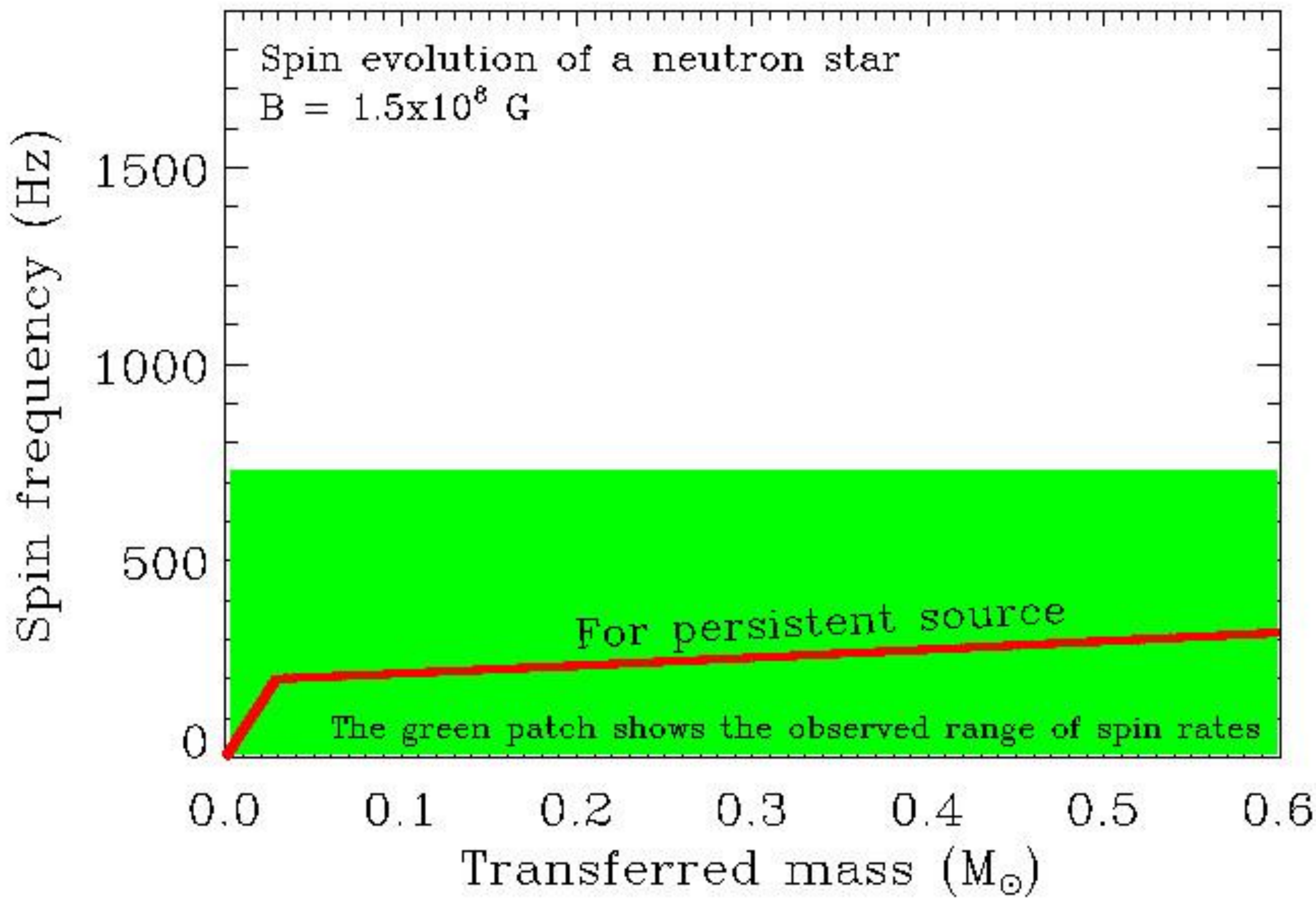
(1) Accretion phase ($r_m < r_{co}$), positive torque on the neutron star (spin-up).

(2) Propeller phase ($r_{co} < r_m < r_{lc}$), negative torque on the neutron star (spin-down).

(3) Quiescent phase, when accretion is stopped by the wind of the pulsar which is turned on.

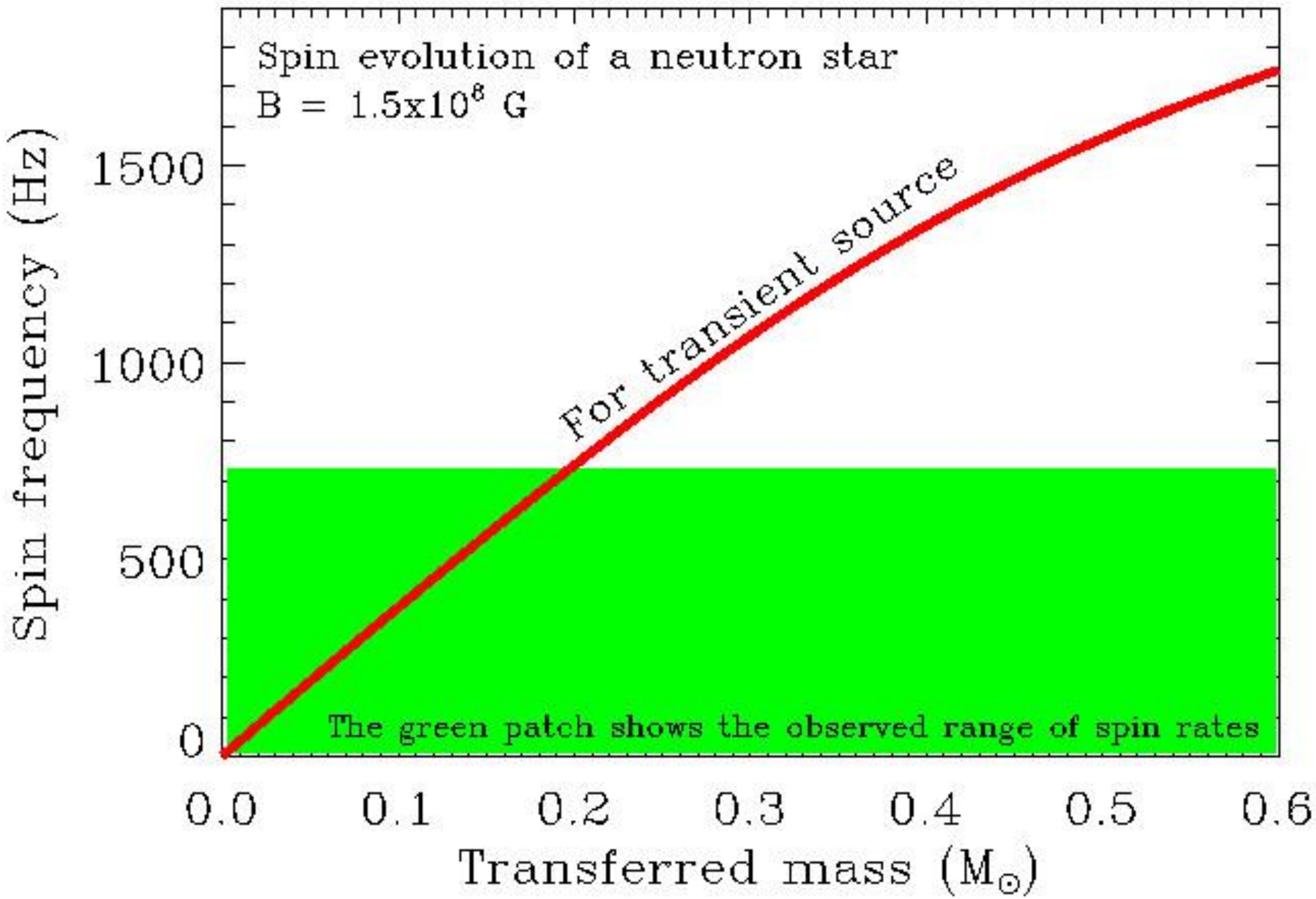
Effect of Transient Accretion on Pulsar Spin-up: numerical computation

An example result from SB & D. Chakrabarty, 2017, ApJ, 835, 4



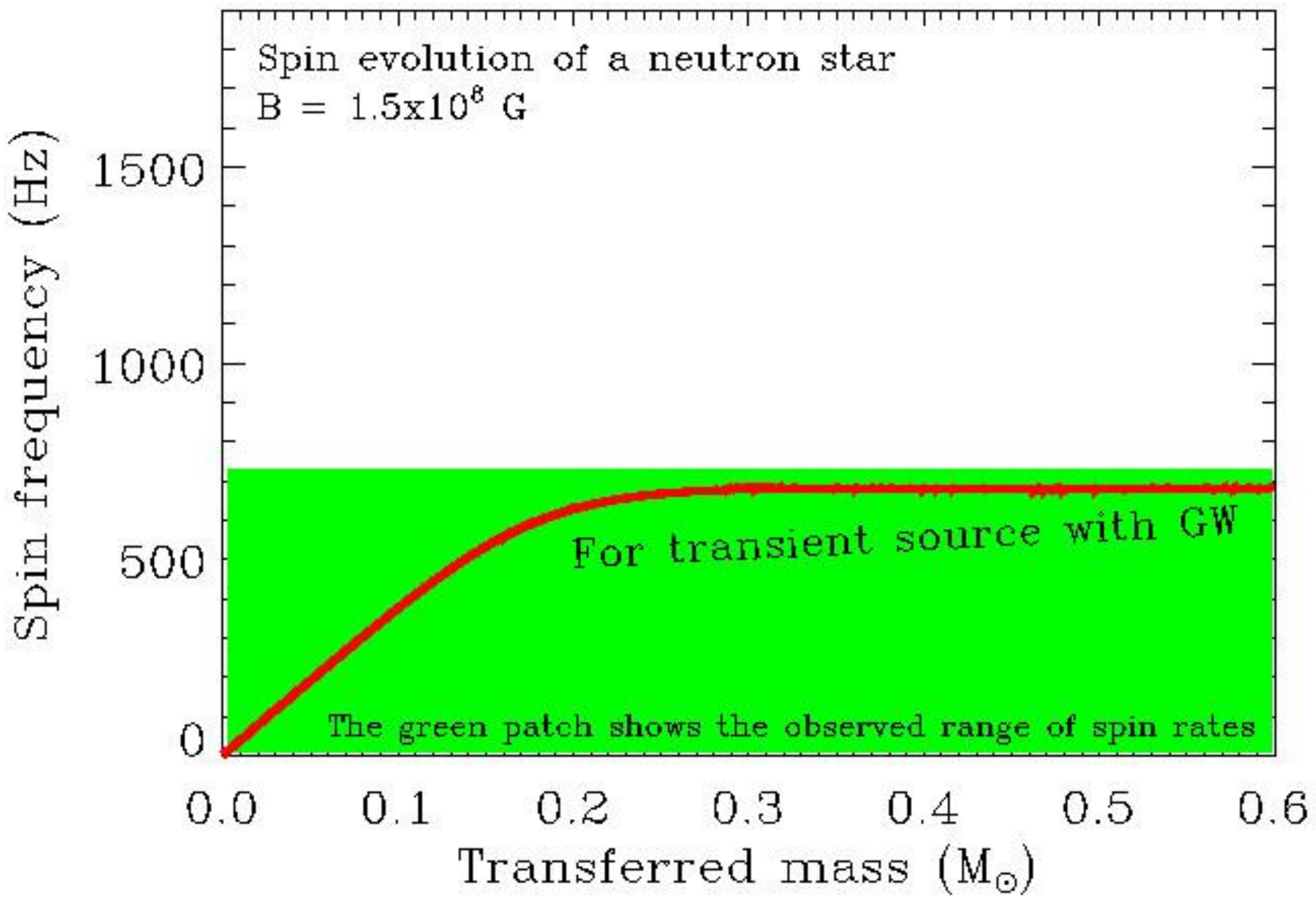
Effect of Transient Accretion on Pulsar Spin-up: numerical computation

An example result from SB & D. Chakrabarty, 2017, ApJ, 835, 4



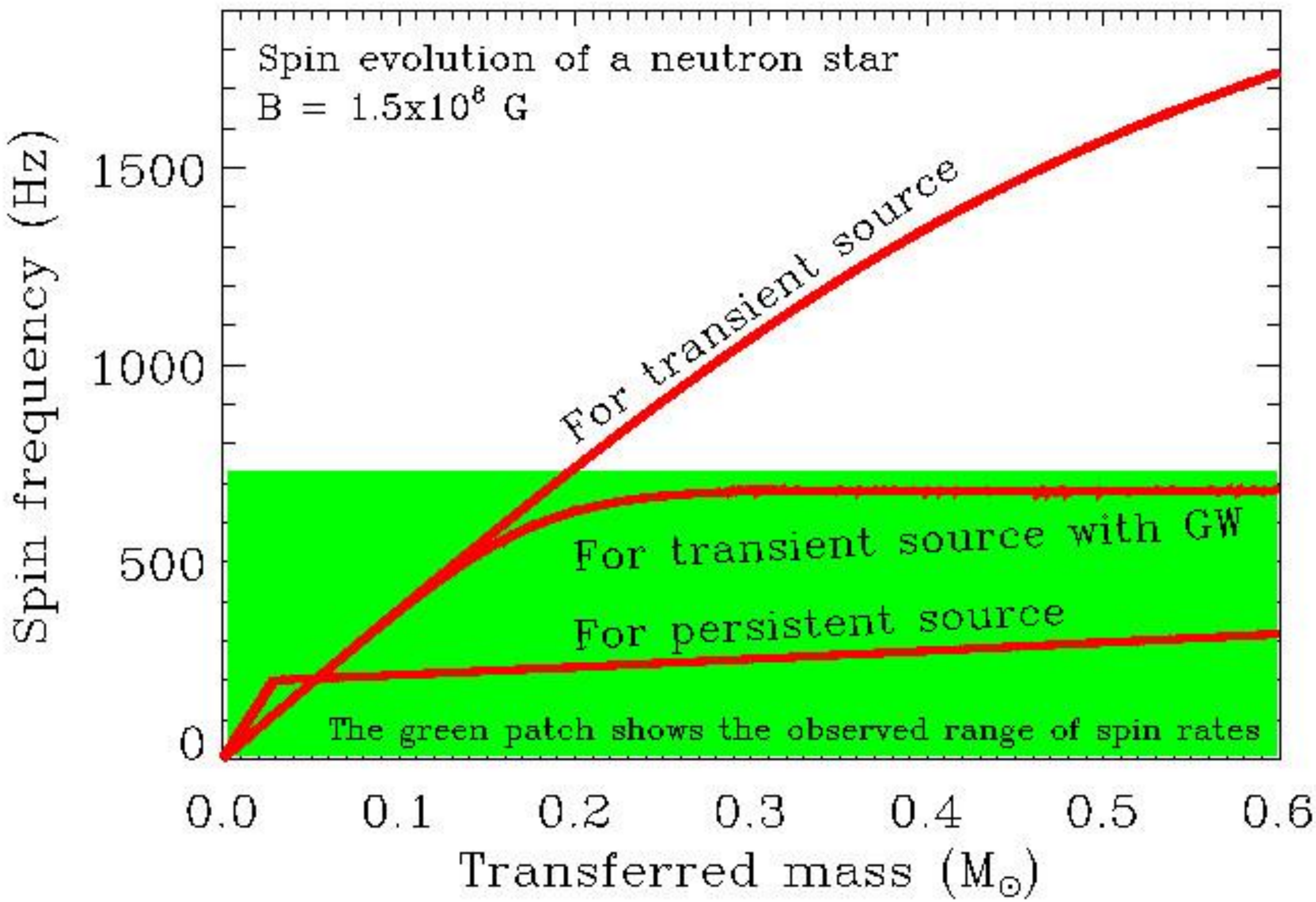
Effect of Transient Accretion on Pulsar Spin-up: numerical computation

An example result from SB & D. Chakrabarty, 2017, ApJ, 835, 4



Effect of Transient Accretion on Pulsar Spin-up: numerical computation

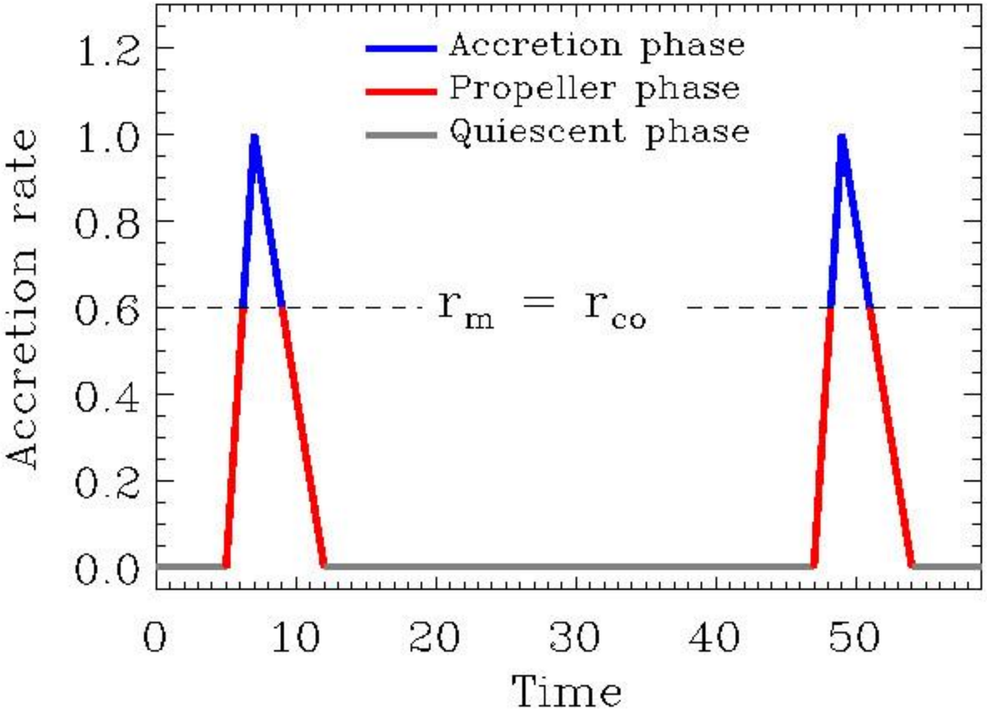
An example result from SB & D. Chakrabarty, 2017, ApJ, 835, 4



So the spin equilibrium frequency for a transient source is higher than that for a persistent source, and the spin equilibrium frequency can be much larger than the observed upper limit of spin rates, indicating gravitational waves from some sources.

What is the spin equilibrium condition for a transiently accreting ms pulsar?

Transiently accreting ms pulsar: spin equilibrium



Magnetospheric radius
$r_m = \xi.[(B^2R^6)/(\dot{M}.(2GM)^{0.5})]^{2/7}$,
where the accretion disk stops.

Corotation radius
$r_{co} = [(GM)/(2\pi\nu)^2]^{1/3}$,
where the Keplerian spin frequency is equal to the NS spin frequency.

In spin-equilibrium. Here too a self-regulated mechanism operates.

For a transient source, the spin equilibrium is reached if total angular momentum transferred to the neutron star is zero during an outburst cycle. The corresponding spin equilibrium frequency ($\nu_{eq,eff}$) is the maximum frequency for a transiently accreting neutron star (because the star will spin down for a higher frequency).

SB & D. Chakrabarty, 2017, ApJ, 835, 4

How do we analytically estimate the spin equilibrium frequency for a transiently accreting ms pulsar?

Analytical calculation of spin equilibrium frequency for transient accretion

Torque on the neutron star : $\frac{dJ}{dt} = \pm \mathbf{A}\dot{M}^{6/7}$

Analytical calculation of spin equilibrium frequency for transient accretion

Torque on the neutron star : $\frac{dJ}{dt} = \pm A\dot{M}^{6/7}$

Total angular momentum transfer : $\Delta J = \int dJ$

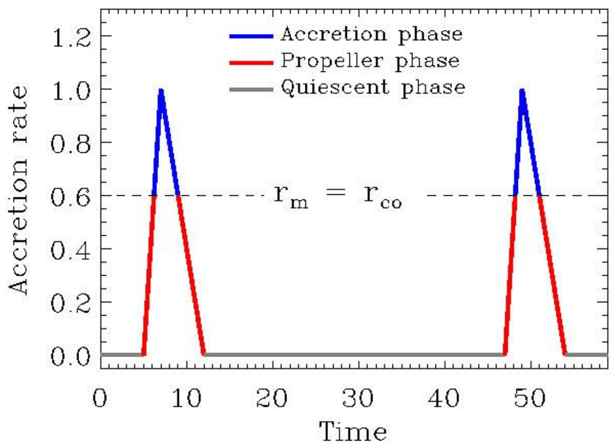
$= +[A\int \dot{M}^{6/7} dt\,]_{Acc} - [A\int \dot{M}^{6/7} dt\,]_{Prop}$

$= +[A_1\int \dot{M}^{6/7} d\dot{M}]_{Acc} - [A_1\int \dot{M}^{6/7} d\dot{M}]_{Prop} = 0$

where (for linear outburst profile),

$A_1 = A/(d\dot{M}/dt) = \text{constant}$

This gives ${\dot{M}_{max}}^{13/7} - {\dot{M}_{eff}}^{13/7} = {\dot{M}_{eff}}^{13/7}$



Here, $\dot{M}_{max}$ is the accretion rate corresponding to the outburst peak, and $\dot{M}_{eff}$ is the accretion rate corresponding to the transition between accretion and propeller phases.

L.H.S. gives the positive angular momentum transfer in the accretion phase, while the R.H.S. gives the negative angular momentum transfer in the propeller phase.

This gives

$\frac{\dot{M}_{eff}}{\dot{M}_{max}} = 0.69$, **which implies,** $\frac{\nu_{eq,eff}}{\nu_{eq,max}} = 0.85$. **[using $r_m = r_{co}$ condition, ∴ $\nu_{eq} \propto \dot{M}^{3/7}$].**

This gives a spin equilibrium frequency ($\nu_{eq,eff}$) expression for transient accretion.

SB & D. Chakrabarty, 2017, ApJ, 835, 4

Summary

- Spin equilibrium frequency provides the basis of the ms pulsar spin distribution study. Almost all the X-ray ms pulsars are transient sources. We give the new concept of spin equilibrium condition and calculate a new expression for the spin equilibrium frequency for transiently accreting ms pulsars.
- We show that the spin equilibrium frequency for a transient source is higher than that for a persistent source, and the spin equilibrium frequency of a transient source can be much larger than the observed upper limit of spin rates.
- Therefore, for the currently inferred values of magnetic field strength and mass accretion rates, we would expect there to be many more rapidly spinning pulsars (even submillisecond pulsars) than we currently detect. This indicates some additional spin-down mechanism, possibly due to gravitational waves.

Summary

- Spin equilibrium frequency provides the basis of the ms pulsar spin distribution study. Almost all the X-ray ms pulsars are transient sources. We give the new concept of spin equilibrium condition and calculate a new expression for the spin equilibrium frequency for transiently accreting ms pulsars.
- We show that the spin equilibrium frequency for a transient source is higher than that for a persistent source, and the spin equilibrium frequency of a transient source can be much larger than the observed upper limit of spin rates.
- Therefore, for the currently inferred values of magnetic field strength and mass accretion rates, we would expect there to be many more rapidly spinning pulsars (even submillisecond pulsars) than we currently detect. This indicates some additional spin-down mechanism, possibly due to gravitational waves.

Thank you !

Disk-magnetosphere interaction

Torques

For the accretion phase:

$$N = \dot{M}\sqrt{GMr_m} + \frac{\mu^2}{9{r_m}^3}\left[2\left(\frac{r_m}{r_{co}}\right)^3 - 6\left(\frac{r_m}{r_{co}}\right)^{\frac{3}{2}} + 3\right]$$

For the propeller phase:

$$N = -\eta\,\dot{M}\sqrt{GMr_m} - \frac{\mu^2}{9{r_m}^3}\left[3 - 2\left(\frac{r_{co}}{r_m}\right)^{3/2}\right]$$

Here, η is an order of unity positive constant, which includes the uncertainty due to unknown fraction of matter ejected for each $\dot{M}$ value.

These expressions can be approximated (with a few percent error) to the following compact form: $N = \pm A\dot{M}^{6/7}$, where A is a positive constant and $+$ and $-$ signs correspond to accretion and propeller phases respectively.

Other torques which we have sometimes used are

EM torque $N_{\mathrm{EM}} \propto \nu^3$

and

GW torque $N_{\mathrm{GW}} \propto \nu^5$

SB & D. Chakrabarty, 2017, ApJ, 835, 4